Columbia University in the City of New York

LAMONT GEOLOGICAL OBSERVATORY
PALISADES, NEW YORK

SOME COMMENTS AND PROSPECTIVES ON DYE DIFFUSION EXPERIMENTS
(Unpublished Technical Note)

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Takashi Ichiye

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1. INTRODUCTION

This note is a result of the author's efforts in groping for some interpretation of dye diffusion data accumulated at the Lamont Geological Observatory since 1961. It is also the purpose of this note to give some ideas on desirable future experiments and to stimulate theoretical study on diffusion in the ocean. Most of the data referred to are described in the unpublished Technical Paper "Dye Diffusion Experiments in the New York Bight" by Costin, Davis, Gerard and Katz (1963), cited as ENY.

2. GROSS FEATURES OF DYE PATCHES

As described in ENY and by Ichiye (1962a), visual, photographic and fluorometric patterns of dye patches show a tendency of elongation in the direction of surface currents (or winds when they are sufficiently strong). There are two possible causes for such elongation. One cause is that the turbulence energy (defined as the mean square fluctuation velocities) is more intense in the direction of the mean current than across the mean current. The other is that the mean current has a vertical shear, usually with a maximum velocity at the surface, and thus the dye concentration at each level spreads with different speeds in the direction of the mean current. In an equili-

brium state, it is believed that turbulence tends to be isotropic. However, there is no reason to generalize an assumption of isotropicity to the surface current generated by transitional wind stresses. Also there is experimental evidence that large eddies in a shear flow are elongated in the direction of the flow (Townsend, 1956). Therefore, the first cause might be a possibility, but experimental verification in the sea and theoretical models of such anisotropic turbulence are yet to come.

The second cause can be analyzed by considering the diffusion equation of the type:

$$\frac{\partial S}{\partial t} + U\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left(A_x\frac{\partial S}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_y\frac{\partial S}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_z\frac{\partial S}{\partial z}\right) \quad (1)$$

in which S is the concentration of dye, U is the mean horizontal velocity varying in the vertical direction Z, and A_x, A_y and A_z are diffusivity in x, y and z directions, respectively. Application of dye technique to determine flow rate and diffusion in a pipe or an open channel was already discussed by Taylor and Elder, using the diffusion equation (1). See the author's review (Ichiye, 1962b) pp. 112-116. However, the flow in a pipe or channel is much simpler than in the sea. There is no reason to believe that the results of experiments in a small hydraulic flume are completely valid in the sea.

Another interesting problem concerning the mean current is the curvature of dye patterns discussed by Costin, Davis, Gerard and Katz

in "Dye Diffusion Experiments in the New York Bight" (1963). They described many examples of dye patches which showed counter-clockwise movement of surface water relatively to the deep water. They estimated this movement from observations indicating that the maximum concentration contained by the surface water moved in most cases cyclonically against the tail part in a curved pattern. They explained such curved patterns from the Ekman spiral in the upper layer of the sea, since effects of change in tidal currents and wind directions with time seem to be too irregular in all series of experiments to produce systematic anticyclonic (from the surface to the bottom) curvature of the hodograph. Their discovery is quite interesting in testing Ekman's theory of wind drift, although possibilities of effects of tidal currents and wind shifts on the curved patterns are still not quite eliminated.

There are two approaches to studying oceanic turbulence, using curved patterns of dye patches, when vertical distributions of currents and dye concentration are known. One approach is to compare observed patterns of dye patches with a solution of the diffusion equation and determine parameters such as eddy diffusivity. One such equation is

$$\frac{\partial S}{\partial t} + U(z)\frac{\partial S}{\partial x} + V(z)\frac{\partial S}{\partial y} = A_x \frac{\partial^2 S}{\partial x^2} + A_y \frac{\partial^2 S}{\partial y^2} \quad (2)$$

in which vertical diffusion is neglected, but x and y, components of mean current, are included, taking into account spiral structures of the wind drift. A solution of equation (2) for the initial source, which is expressed by $F(x, y, z)$ is given by:

$$S(x,y,z,t)=\frac{1}{4\pi\sqrt{(A_x A_y)}\,t}\int_0^\infty\!\!\int_0^\infty F(\alpha,\beta,z)\exp\left[-\frac{(x-Ut-\alpha)^2}{4A_x t}-\frac{(x-Vt-\beta)^2}{4A_y t}\right]d\alpha\, d\beta \tag{3}$$

When the vertical mixing is considered, the solution of a diffusion equation becomes more complicated than equation (3), but it can be obtained either numerically or by successive approximation from (3). However, there is no need to determine the exact solution, because the diffusion equation itself is an approximation. Further, the quantities appearing in the diffusion equation can be measured in the sea only with limited accuracy. In most cases of the experiments, only the solution for a vertical line source, which is given by $F(z)$, is sufficient for determining order of magnitudes of eddy diffusivity. The concentration in aerial photographs of dye patches is represented by an integral:

$$\int_0^\infty S(x,y,z,t)\,e^{-kz}\,dz \tag{4}$$

in which k is a vertical extinction coefficient for visibility of dye in the water, and z-axis is taken downwards from the surface. The coefficient k is to be determined from experiments, in which dye patches of known concentration at different depths must be photographed from an airplane.

Another approach to oceanic turbulence by using curved patterns is to determine vertical eddy viscosity in the upper layer. Suppose that dye is initially distributed uniformly along a vertical axis.

If there is a horizontal current whose direction and speed are variable with depth, the initial dyed filament of water is distorted. If there is no diffusion, projection of the filament on a horizontal plane represents a hodograph (of the trajectory), and it forms a spiral for wind drifts corresponding to Ekman's theory. The projected curve can be expressed by a position vector $\vec{r} = (\xi, \eta)$ (5) defined by:

$$\frac{d\xi}{dt} = u(x, y, z, t) \; ; \; \frac{d\eta}{dt} = v(x, y, z, t) \quad (6)$$

in which the initial values of ξ and η are taken at the origin of coordinates. The curvature R^{-1} of the projected curve of the filament (called hereafter simply the dyed curve) is given by a function of the distance s measured from an arbitrary point along the curve, such as:

$$\frac{1}{R(s)} = \left| \frac{d^2 \vec{r}}{ds^2} \right| \quad (7)$$

according to a theorem of differential geometry, where:

$$ds^2 = d\xi^2 + d\eta^2 \quad (8)$$

In order to express the curvature by functions of a parameter Z instead of s, the following relations are used:

$$\frac{d^2\vec{r}}{ds^2} = \frac{d^2\vec{r}}{dz^2}\left(\frac{dz}{ds}\right)^2 + \frac{d\vec{r}}{dz}\frac{d^2z}{ds^2} \quad (9a)$$

$$\frac{dz}{ds} = (\xi_z^2 + \eta_z^2)^{-\frac{1}{2}} \tag{9b}$$

$$\frac{d^2z}{ds^2} = -(\xi_z \eta_{zz} + \eta_z \xi_{zz})(\xi_z^2 + \eta_z^2)^2 \tag{9c}$$

in which the subscript z indicates differentiation with z. Substitution of (9b) and (9c) into (9a) yields:

$$R(z)^{-1} = |\eta_z \xi_{zz} - \xi_z \eta_{zz}| \; (\xi_z^2 + \eta_z^2)^{-3/2} \tag{10}$$

In a stationary and horizontally uniform current, ξ and η becomes:

$$\xi = u(z)t \quad ; \quad \eta = v(z)t \tag{11}$$

and equation (10) can be expressed in terms of horizontal velocity. When the current is generated from balance of Coriolis' force and shearing stresses, as in Ekman's wind drift, the equations of motion are:

$$-fv = \frac{d}{dz}\left(K\frac{du}{dz}\right) \quad ; \quad fu = \frac{d}{dz}\left(K\frac{dv}{dz}\right) \tag{12 a \& b}$$

in which f is the Coriolis' coefficient and k(z) is the eddy viscosity variable with depth. Substituting relations of u_{zz} and v_{zz} of (12 a & b) into (10), we have:

$$K(z) = \frac{f R(z) \frac{d}{dz}(u^2 + v^2)}{t\;(u_z^2 + v_z^2)^{3/2}} \tag{13}$$

Equation (13) indicates that the eddy viscosity at each level can be determined when the curvature of the dyed curve and velocity and shear of the mean currents are known. It is noted that the eddy viscosity is proportional to the radius of curvature. When the longer central line of an elongated pattern is interpreted as the hodograph of the wind-driven current, the example of the patch in figure 9 of ENY gives the following numerical values:

$$f = 10^{-4}\ (\mathrm{sec}^{-1})\ ;\ \bar{R} \approx 10^{5}\ (\mathrm{cm})$$

$$u \sim v \sim 10\ (\mathrm{cm/sec})\ ;\ u_z \sim v_z \sim 10^{-2}\ (\mathrm{sec}^{-1})$$

$$t = 10^{4}\ (\mathrm{sec})$$

Equation (13) gives the averaged value with depth of K equaling 100(cm^2/sec), which is not unreasonable compared with other estimates of vertical eddy viscosity. In more general cases of currents which are non-stationary and driven by pressure gradients as well as winds, an equation for k(z) similar to (13) can be obtained by substituting equations of motion into (10), because the term containing k'(z) vanishes in the right-hand side of (10). However, since the resulting relation has so many terms which are not easily estimated from observation, such elaboration is hardly justifiable.

3. APPLICATION OF ISOTROPIC DIFFUSION MODELS

Data on dye diffusion were very poor in their quality a few years ago, since there was no continuously recording type fluorometer and

even good aerial photographs were scarce. In order to obtain a structure of turbulence from such data, the author (Ichiye, 1959) determined change of areas with dye concentration above a threshold value using visual observations of dye patches and applied Taylor's theory of diffusion by discontinuous movements and a semi-empirical theory of Joseph and Sendner, assuming that the diffusion of dye patches is horizontally isotropic. Okubo (1962) elaborated this approach and tried to verify various diffusion models (isotropic) by curve-fitting techniques. However, such a method of verification is almost meaningless, since any curve may fit (or deviate from) few, scattered points of observed data of dubious quality. (This does not mean that theoretical study on diffusion process is meaningless, though.)

Many aerial photographs of high quality, combined with concentration contours determined with fluorometers, indicate that details of dye patches with elongated, finger-like or striated shapes are quite different from the features predicted by a simple theory of isotropic diffusion. Also, as indicated in ENY, other environmental factors suggest that the shape of these patches is dependent on advective currents and vertical diffusion, as discussed in the previous section, or on instability of shearing flow, as will be discussed in the next section. It is the zeroth order approximation to the diffusion process to apply results of conventional theories on turbulent diffusion to dye patterns assumed as circumsymmetrical and derive

turbulence characteristics by curve fitting. The next approximation must be such that the patterns of dye patches may be correlated with direct measurements of mean currents, their fluctuations, stability and other environmental data.

4. STRIATED STRUCTURES OF DYE PATCHES

Many examples of dye patches described in ENY showed striated patterns, in which a rather regular array of furrows is, in most cases, almost parallel to the wind direction. In some cases, when the dye patch was elongated and curved, as described in section 2, the outlook of the striated pattern superimposed on the patch is similar to the pattern of flows caused by roll-type instability in rotating tank experiments of Faller (1963). However, such similarity might be only superficial, because scales of streaks and time period of their development are so small in the natural water that effects of Coriolis' terms seem to be unimportant to generation of such streaks. But Coriolis' force might be effective when the wave lengths (or interval between the furrows) of streaks are large and the striated pattern is maintained for a long period. To check such effects theoretically is a challenging problem for geophysical hydrodynamicists.

Mechanism of generation of striated patterns seems to be quite intriguing. If it is assumed that the furrows are due to a roll-type cellular motion caused by instability in a uniform surface current with vertical shear, such motion must be generated by non-linear interaction between the mean current and perturbations. This can be easily

seen from linearised perturbation equations for a shear flow discussed by Lin (1955). If the wave velocity of perturbations have a component moving parallel to the mean current, stability of the current can be discussed as a linear perturbation problem as treated by Lin. However, if the perturbations are variable only in the directions perpendicular to the mean current, the linearized perturbation equations yield only gravity waves (with damping) with crests parallel to the mean current. Therefore, in order to determine the stability of the shear current for the long perturbations parallel to the current as in cellular motion which is assumed to cause furrows, non-linearity must be considered. This is a reason that mathematical formulation for flows responsible for Lagmuir streaks is not achieved yet.

Welander (1963) proposed a speculation on mechanism of natural streaks on the sea surface. He assumed that material forming a surface film of the sea might be concentrated by chance along a band in the direction of the wind, even when the wind is uniform. Such material causes damping of capillary waves in the band. The wind becomes stronger along the band due to decrease of surface friction than on both sides of it. Such difference in friction will also create a secondary cellular circulation perpendicular to the mean wind, and the air close to the surface flows toward the band. Such air motion in turn will produce cellular motion of the water.

Welander's hypothesis seems to be novel, because the mechanism is sought in the air motion rather than in the water motion, and also the

importance of a surface film is assumed. However, his hypothesis cannot explain the regularity of the intervals of furrows observed in dye patches. In order to test his speculation and, at the same time, to determine fine structures of the streaks, three measurements, which are feasible with relative ease, can be suggested in connection with observations of dye patches. The first is to measure fine structures of wind over the sea by using a smoke signal. The second is to take underwater pictures (movie or still) of rear views of striated patches, either by a skindiver or with an underwater TV camera. The third is to measure in situ surface tension of the water or to change it artificially by adding chemicals to the sea surface.

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